A Theory of Cost-Sharing Renegotiations in Military Alliances

Yuji Masumura*

Last Updated: June 24, 2025

The literature on military alliances suggests that alliances can deter aggression through costly signaling. In reality, however, protégés often share substantive alliance costs, which makes alliances cheaper for a patron and should derail the signaling. Why do protégés do that? To explain this gap, I develop a formal model in which allied countries renegotiate their cost-sharing under the shadow of international crisis and domestic politics. The model identifies two means by which cost-sharing negotiations sustain peace. First, successful negotiations keep a patron's involvement by reducing the alliance costs when the patron is *not* strongly committed. Second, a large cost-sharing demand makes the negotiations fail, but it signals the patron's commitment and panders to domestic isolationism at the same time. Empirical records of the US-Japan alliance in 1978 and 2019 are explained through these mechanisms.

*University of Texas at Austin, email: yuji.masumura@utexas.edu. The author would like to thank Scott Wolford, Terry Chapman, Pat McDonald, Jesse Johnson, participants at EITM 2024, APSA 2024, and Texas Triangle 2025 for helpful comments and suggestions. The author also thanks Mozart's Coffee Roasters, Medici Roasting, and Hold Out Brewing in Austin for inspiring work environments.

1 Introduction

In November 2019, the Trump administration reportedly asked Tokyo to pay four times as much, even though it already annually paid "$2 billion to offset the cost of 54,000 U.S. troops" (Seligman and Gramer, 2019). Along with Trump's similar complaints about the "delinquent" contribution of other US allies, the administration evoked questions about the US commitment to its alliances all over the world (Hille et al., 2020). Trump would be unique in his way of delivering the demand, but the cost-sharing renegotiations between the U.S. and its allies are not new at all. In fact, the past US administrations were dissatisfied with the original agreements and requested their allies to shoulder more costs of their alliances, and some of the renegotiations were successful.

According to the current literature, successful cost-sharing by a protégé *should* weaken deterrence because alliances are often regarded as a costly signaling device (Morrow, 1994, 2000; Smith, 1995). The idea is that, if a patron sinks enough costs, the patron can signal its intention to help its protégé in a war. In other words, a patron who does not invest in its alliance appears to be less committed and fails to deter aggression. If this understanding is true, successful cost-sharing between allies makes their alliances cheaper for a patron, and it should derail signaling and deterrence.

Politicians, however, seem to have different opinions. They often claim the positive effect of cost-sharing on deterrence. For example, right after reaching an agreement about cost-sharing of the US Forces Japan with the Biden administration, a Japanese Foreign Minister expressed his view, " reaching to a necessary agreement at the early stage of the Biden administration shows both countries' strong commitment in the US-Japan alliances,

increases the credibility of the alliance, and dispatches it to the international society[1].” A similar view is shared by the United States in its 2017 joint statement, saying the cost-sharing by the Japanese government “ serves as a pillar of the Alliance and a symbol of Japan’s enduring support for the U.S. military presence in Japan” [2]. There seems to be a gap between what practitioners say and think and what scholars understand. Are the politicians’ claims true? If so, by what mechanism does alliance cost-sharing improve deterrence? The literature does not have an answer to these questions[3].

Cost-sharing is even more puzzling since such cost-sharing does not seem to directly improve deterrence. For example, Japan and South Korea pay some costs of the US bases in their countries such as salaries to general workers, utilities bills, and construction fees for houses, golf courses, and movie theaters that are exclusively for US soldiers[4]. It is hard to imagine that these costs directly contribute to deterrence. In this respect, cost-sharing is different from burden-sharing, which is about a protégé’s arms buildup and the costs are

[1]Extraordinary Press Conference by Minister Toshimitsu Motegi on February 17th, 2021. Website of Ministry of Foreign Affairs (URL: https://www.mofa.go.jp/mofaj/press/kaiken/kaiken4_001029.html Last Access: November 25, 2023). translated by the author

[2]Joint Statement of the Security Consultative Committee, The website of US Department of State (URL: https://2017-2021.state.gov/joint-statement-of-the-security-consultative-committee/index.html Last Access: November 25, 2023)

[3]Taking reputation costs of abandoning an ally into account would make this question more puzzling. Such costs would make it more difficult for an ally to ask for cost-sharing, and thus, cost-sharing should be a clear sign of being uncommitted.

[4]For South Korea, see pp. 154-156 of 2022 White Paper of the Ministry of National Defense of Republic of Korea (URL: https://www.mnd.go.kr/cop/pblictn/selectPublicationUser.do?siteId=mndEN&componentId=51&categoryId=0&publicationSeq=1057&pageIndex=1&id=mndEN_0313000000000 Last Access: November 25, 2023). For Japan, see pp. 379 -281 of 2023 White Paper of the Ministry of Defense of Japan (URL: https://www.mod.go.jp/en/publ/w_paper/index.html Last Access: November 25, 2023) and the website of the Ministry of Defense of Japan (URL: https://www.mod.go.jp/j/approach/zaibeigun/us_keihi/ Last Access: November 25, 2023). They are often called Host Nation Support in US government documents, but it is also called cost-sharing in Japanese or South Korean documents, as well as Omoiyari Yosan (sympathy budget) in Japanese.

used for directly improving a protégé's military capability (Iwanami, 2023; Blankenship, 2021, 2023, 2020)[5]. Why do some US allies agree to pay the costs of their alliance as a result of renegotiation, even though it does not directly improve their power and should weaken the signaling?

To account for the questions above, I develop a three-actor formal model in which allies have a cost-sharing renegotiation under the shadow of international crisis and domestic politics. Unlike most existing models, I allow endogenous cost-sharing by allies facing an external threat of aggression. Specifically, in the model, the alliance cost-sharing affects (a) the costs and survival of the alliance, (b) the reward from domestic constituencies for a patron, and crucially (c) the success of signaling. The patron starts a new cost-sharing negotiation with its protégé. This renegotiation determines who pays how much, but it also entails rewards/punishment by domestic constituencies and the danger of the patron's withdrawal from the alliance. After the negotiation, they enter crisis bargaining with a potential aggressor who is unsure about how much the patron shares its preferences with its protégé. The potential aggressor observes the result of the negotiation and updates its belief, so the allies have to make decisions at the cost-sharing negotiation stage in consideration of the later crisis bargaining.

The model reveals that a credible threat of abandonment is key to successful cost-sharing renegotiations. A protégé does not agree to share additional costs of the alliance unless a patron's threat to withdraw from the alliance is credible. Put differently, successful cost-sharing renegotiations happen only when the patron would withdraw from the alliance

[5]Burden-sharing renegotiations are what the US and other NATO countries have often contended over. Such renegotiations would be less counterintuitive than cost-sharing since these costs directly improve the protégé's deterrence, though they may not be good for signaling. So, allies can collectively benefit from increased capability as a result of burden-sharing.

otherwise.

Based on this insight, the model identifies two means by which cost-sharing renegotiations sustain peace. First, successful renegotiations keep a patron's participation in the alliance by reducing the alliance costs when the patron is *not* strongly committed. Given that successful cost-sharing renegotiations occur in equilibrium only when a threat of withdrawal is credible, such success is a sign that the patron does not highly value the issue at stake, and this information is revealed to a potential aggressor. However, a successful cost-sharing renegotiation also makes the alliance cheaper for a patron, and this makes it easier for her to maintain the alliance. Keeping the alliance brings capability advantages, which sustains a high probability of winning. That way, cost-sharing brings deterrence and peace. In other words, cost-sharing is successful because the capability advantages of the alliance are prioritized over the signaling aspect of the alliance, which implies a potential tension between signaling and capability in alliances. This equilibrium is also consistent with Morrow (1991), which argues that protégés pay autonomy costs to enjoy security benefits in asymmetric alliances. But, unlike Morrow (1991), this equilibrium specifies what "autonomy costs" are and how the exchange rate is determined.

Second, a large cost-sharing demand makes the renegotiations fail, but it signals the patron's strong commitment and panders to domestic isolationism at the same time. When a patron is committed to the alliance, a threat of withdrawal becomes incredible, eliminating a protégé's incentive to accept a cost-sharing offer. Since cost-sharing does not succeed whatever the patron offers, she maximizes her payoff by pandering to her domestic constituencies, which leads to a high cost-sharing demand when they are isolationists. This large demand is beyond the acceptable range for the protégé, and thus, the cost-sharing

negotiation is doomed to fail. However, a potential aggressor rationally conjectures that such failure is a sign of the protégé's confidence in its patron's support. Thus, a failed renegotiation, coupled with a large demand, signals the strong commitment of the patron, leading to peace and successful deterrence. This equilibrium suggests that there emerges a *positive* correlation between a patron's commitment and a cost-sharing demand because of the patron's domestic isolationists. In other words, a cost-sharing demand becomes large because the patron tries to secure *both* international security and domestic reward.

Based on the model, I account for empirical records of cost-sharing negotiations in the US-Japan alliance in 1978, when Japan started the cost-sharing with the United States, and in 2019, when Japan rejected a cost-sharing offer from the Trump administration. These case studies successfully show important similarities between the model's implications and the empirical evidence.

From the perspective of alliance management, this paper also sheds light on how allies manage to get through changes in their strategic environment and maintain their deterrence. Studies show that large changes in power are strongly associated with dishonoring their alliance commitments (Leeds, 2003a), and changes in preferences are the main reasons for alliance termination (Leeds and Savun, 2007). Descriptive statistics also show that new military alliances have been rare recently and existing military alliances are getting older (Kenwick and McManus, 2021), which suggests that the existing alliances survived changes in their strategic environment, such as the end of the Cold War or the Vietnam War. This article explains why some alliances have survived these environmental changes through cost-sharing. Overall, the model shows that, in equilibrium, both successful and failed cost-sharing renegotiations can be beneficial for deterrence, but in different ways: successful

renegotiations sustain an alliance, and thus, allies keep high aggregated capability even when a patron is *not* strongly committed. Failed renegotiations, despite a high cost-sharing demand, reveal a protégé's confidence in a patron's strong commitment and pander to a patron's strong domestic isolationism. This article provides important implications of cost-sharing renegotiations in military alliances. Cost-sharing negotiations work as a regulator valve both when a patron is committed and not committed, and thus, cost-sharing allows alliances to survive longer by overcoming changes in the strategic environment.

2 Literature

The main contribution of this article to the literature is to cast light on the signaling aspect of cost-sharing renegotiations between allies. Morrow (1994, 2000) and Smith (1995) all regard alliances as costly signaling and explain why alliances deter aggression through this mechanism. This view is probably the most canonical and current understanding of alliances, as shown by the fact that many empirical studies and literature reviews employ this as a source of the deterrence effect of alliances (Leeds, 2003b; Johnson and Leeds, 2011; Leeds and Anac, 2005; Morrow, 2000; Kenwick and McManus, 2021). However, these studies do not pay attention to (asymmetric) cost-sharing of alliances, meaning cost-sharing negotiations, especially protégé states' influence on them, are not recognized in the signaling process. By measuring peacetime costs using treaty designs in the ATOP project (Leeds et al., 2002)[6], Johnson (2022) empirically finds that peacetime costs are larger when allies face a stronger challenger, suggesting that the amount of peacetime

[6]To wit, scholars find it not easy to directly measure the costs of alliances, as seen in the discussion between Alley and Fuhrmann (2021) and Cooley et al. (2022).

costs of alliances are selected "carefully with their adversaries in mind". This is one of the most important papers studying peacetime costs, but peacetime costs are assumed to be determined as a function of a potential challenger, not intra-alliance politics, meaning cost-sharing (re)negotiations between allies are out of the scope. Similarly, Johnson (2015) studies the interaction of a protégé's concession and alliance formation, but does not focus on how existing alliances adjust their cost-sharing after alliance formation and its effect on crisis bargaining.

Other existing studies investigate alliance burden-sharing, not cost-sharing (Blankenship, 2021, 2023; Becker et al., 2023; Blankenship, 2020; Iwanami, 2023). They regard burden-sharing as coordinated military capability among allies and focus on how much allied countries arm themselves in response to their partner's request, sometimes under the shadow of aggression (Iwanami, 2023). However, these studies do not explicitly focus on how signaling can happen in the context of burden-sharing. Although in a slightly different context, this article focuses on the signaling aspect of cost-sharing (re)negotiation by using a formal model, which I will explain in the next section.

3 Model

The model has three actors, Ally (A), Target (T), and Challenger (C). I call A she, T he, and C it to avoid confusion. A and T have an alliance at the beginning of the game, and A is supposed to intervene when T is attacked. T and C have a conflict over an issue space that T holds, such as territory, of which value is 1 for both T and C and is $\beta \in (0, 1)$ for A. β can be thought of as A's bias over the issue space, or how much A shares T's preference.

Imagine a situation where some shock to A's preferences (β) occurred after the alliance formation, and this brings uncertainty about A's commitment to the alliance. Changes in the strategic environment (e.g., the end of the Vietnam War or the Cold War), economic depression (e.g., the 2008 financial crisis and the US's subsequent Rebalance to Asia), and leadership turnover (e.g., the inauguration of Donald Trump)[7] are examples of such shock. To represent this situation, I assume C' uncertainty about A's value of the issue (β). The game starts with Nature (N) randomly choosing A's type; a high type ($\overline{\beta}$) with the probability of $\phi \in (0,1)$ and a low type ($\underline{\beta}$) with the probability of $1-\phi$. This information is revealed to A and T, but not C[8].

Following Morrow (1994), the alliance has two effects. First, the alliance is costly. In the model, the total costs of the alliance are denoted as $\pi > 0$, and A pays the entire costs at the beginning of the game. These costs are necessary to work as costly signaling and can be understood as peacetime costs in Morrow (1994). The costs of the alliance, π, can entail various things, including a shared military plan in case of war (Poast, 2019; Johnson, 2022), the costs of overseas military deployment and joint military exercises, and the restriction of foreign policy autonomy (Morrow, 1991). Second, the alliance increases the probability of winning if allies fight together (Morrow, 1994). By keeping an alliance, states find it easier to have integrated joint military plans (Poast, 2019; Johnson, 2022) and gain public support (Tomz and Weeks, 2021; Tomz et al., 2020, 2023). Thus, I assume that A and T win with a higher probability if they keep their alliance *and* fight together. Specifically, the probability of winning is denoted as $p \in (0,1)$, and it takes p_I if T fights

[7] See Wolford (2007)

[8] No uncertainty between allies is a common assumption in the literature, such as Fang et al. (2014). See Smith (2021) for a model where a country has uncertainty about its coalition partner.

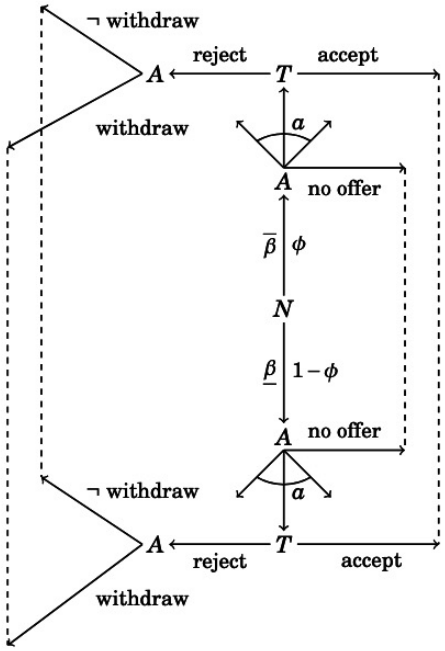

Figure 1: Cost-Sharing Negotiation Stage

alone (For example, Poland fought alone against Nazi Germany in 1939, even though it had an alliance with France and the United Kingdom), p_m if T and A fight together without the alliance (The United States and Kuwait fought together against Iraq in 1990 without an alliance), and p_h if they fight together with the alliance (Germany and Austria-Hungary had an alliance and fought together against Russia in 1914). Note that $p_l < p_m < p_h$. The difference between p_m and p_h is called the capability advantages an alliance brings.

The model has two stages; the cost-sharing (re)negotiation stage and the crisis bargaining stage. Figure1 shows the cost-sharing negotiation stage. In this stage, after A's type is realized, she chooses to make either "no demand" or a cost-sharing offer. In the former case, the cost-sharing negotiation does not happen, and A pays the full costs of the alliance (π), meaning no change in the cost-sharing. In the latter case, she demands T to pay some

alliance costs, and the amount of the offer is denoted as $a \in (0, \overline{a}]$, where $\overline{a} > 0$ is the upper limit of T's ability to pay. I do not presume the way a is paid, but the prime example is Japan and South Korea's monetary contribution to the US forces stationed in each country.

Then, T decides whether to accept the offer. Accepting the offer is a success of the cost-sharing renegotiation and this imposes a on T and $\pi - a$ on A. Rejecting the offer is a failure of the renegotiation. This does not impose any costs on T but gives A an opportunity to withdraw from the alliance. If A withdraws from the alliance, both A and T do not pay any costs, but they cannot benefit from the alliance capability advantages[9]. If A does not withdraw from the alliance, she pays the full cost of the alliance, π. In sum, the alliance cost-sharing renegotiation has four outcomes: no cost-sharing renegotiation, which I call "no demand", the success of the renegotiation, which is "sharing", the failure of the renegotiation with A remaining in the alliance, which is "free-riding", and the failure of the renegotiation without the alliance, which is "withdrawal".

Finally, I incorporate elements of domestic politics by simply allowing A to get la when she demands something in the negotiation. The domestic politics term, $l \in (-1, \infty)$, represents the level of domestic isolationism/internationalism in A, and it brings punishments/rewards for a harsh attitude towards its ally. When $l > 0$, the domestic constituencies are isolationists on average and reward a leader's harsh attitude towards allies by giving a positive value ($la > 0$) to the leader, for example, by re-electing the leader with a higher probability. When $l < 0$, the domestic constituencies are internationalists and punish a leader who demands a lot from their allies by giving a negative value (e.g. not re-electing

[9]The interpretation of the withdrawal from the alliance is flexible, including many things from formal alliance termination to partial withdrawal. This does not change the implications of the model as far as allies lose the capability advantages.

the leader)[10]. The lower limit of l eases the analysis, but substantively, it removes cases where A does not want to get a concession from her ally, even though it's possible, due to her extreme domestic internationalism. Such cases are out of the scope of this article.

I denote the payoffs resulting from the cost-sharing stage as $\lambda_{i,n}$, where i is the player and can take either A or T ($i = \{A, T\}$) and n is the result of the negotiation and can take one of the four outcomes of the negotiation ($n = \{\text{no demand}, \text{sharing}, \text{free-riding}, \text{withdrawal}\}$). The payoffs from the negotiations are formally expressed below.

$$\lambda_{A,n} = \begin{cases} -\pi \quad (\text{if } n = \text{no demand}) \\ -\pi + a + la \quad (\text{if } n = \text{sharing}) \\ -\pi + la \quad (\text{if } n = \text{free-riding}) \\ 0 + la \quad (\text{if } n = \text{withdrawal}) \end{cases} \tag{1}$$

$$\lambda_{T,n} = \begin{cases} -a \quad (\text{if } n = \text{sharing}) \\ 0 \quad (\text{if } n = \text{free-riding, no demand, or withdrawal}) \end{cases} \tag{2}$$

Next, the model moves on to the crisis bargaining stage. In this stage, C first proposes a division of the one-unit policy space to T. The proposal is expressed as $x \in (0, 1)$, where x is what T keeps and $1 - x$ is what C obtains. After receiving the demand offer, T chooses to

[10]Note that these rewards/punishments are for the *attitude* of the leader, not for the result of the negotiation itself. Harsh attitudes towards the cost-sharing issue, even if they are not successful, correlate with the leader's attitude toward other diplomatic issues, such as international organizations or trade barriers, and I assume that the domestic constituencies reward or punish these attitudes to discipline leaders' future behavior. Alternatively, we can interpret l as a leader's personality that gives the leader a positive or negative value for demanding a lot from an ally, and domestic constituencies elect a leader that fits their preference. This does not change the interpretation of the results. In any case, this way of representing domestic politics is more appropriate than hand-tying in the context where strong partisanship immunizes a leader from a negotiation failure or inconsistency (See McDonald et al., 2019).

concede or resist. If he concedes, the game ends with a settlement and C's offer is realized. If T rejects, a war between T and C breaks out. Then, A chooses whether to help T or not. If A helps T, the war expands to a multinational war. If A does not help, then the war remains bilateral. War payoffs are expressed as a costly lottery, and actors have to pay the war costs (c_i) if they are involved. The final payoffs are shown below.

$$u_i(\text{Settlement}) = \begin{cases} \beta x + \lambda_{A,n} & (\text{if } i = A) \\ x + \lambda_{T,n} & (\text{if } i = T) \\ 1 - x & (\text{if } i = C) \end{cases} \tag{3}$$

$$u_i(\text{Bilateral War}) = \begin{cases} \beta p_l + \lambda_{A,n} & (\text{if } i = A) \\ p_l - c_T + \lambda_{T,n} & (\text{if } i = T) \\ 1 - p_l - c_C & (\text{if } i = C) \end{cases} \tag{4}$$

$$u_i(\text{Multilateral War w/ alliance}) = \begin{cases} \beta p_h - c_A + \lambda_{A,n} & (\text{if } i = A) \\ p_h - c_T + \lambda_{T,n} & (\text{if } i = T) \\ 1 - p_h - c_C & (\text{if } i = C) \end{cases} \tag{5}$$

$$u_i(\text{Multilateral War w/o alliance}) = \begin{cases} \beta p_m - c_A + \lambda_{A,n} & (\text{if } i = A) \\ p_m - c_T + \lambda_{T,n} & (\text{if } i = T) \\ 1 - p_m - c_C & (\text{if } i = C) \end{cases} \tag{6}$$

4 Key Features of the model

The model has some key features. First, building on existing works, the costs of alliances are sunk (Morrow, 1994, 2000). Scholars often regard the formalization of alliances as the source of their deterrence effects. States write down the terms of alliances and make them open to the public because states can generate some sunk costs (Morrow, 2000), which is essential to work as costly signaling (Fearon, 1997). In my model, A and T have to pay the costs of the alliance regardless of the results of the crisis bargaining, satisfying the condition for sunk costs.

Second, the depth of cost-sharing (a) does not directly affect military capabilities. This is because it is hard to imagine an automatic relationship between the alliance costs T pays and the improved military capability, where the costs are used for utility bills, houses, amusement facilities, and so on. If there is a relationship between the depth of cost-sharing and military capability, there must be some political process that converts the alliance costs to the military capability between allies, and this is exactly what the model tries to capture.

Third, the total costs of the alliance, π, are given and fixed in the model. Scholars often assume that the alliance costs, such as peacetime costs or the level of institutionalization, are determined at the alliance formation stage (Leeds and Anac, 2005; Johnson, 2022; Morrow, 1994). As my focus is on how allies renegotiate the costs of their *existing* alliance, I set π as a fixed value. One can interpret this parameter as the optimal level of costs to deter an aggressor in the past or the fixed costs of the investment in international institutional building, but allies have a conflict over who pays how much.

Fourth, and most importantly, I represent the domestic politics by rewarding/punishing A for the amount of the cost-sharing demand through a simple parameter, l. As we will

see, this parameter is important to differentiate a committed patron's strategies at the cost-sharing stage: If we do not have this parameter, a committed patron who cannot generate a credible threat of withdrawal is indifferent about the result of the negotiation, leading to the inability to discipline her cost-sharing offers on equilibrium.

The model is different from existing formal models in the following points. Morrow (1994) investigates how the peacetime costs of alliances, coupled with improved military performance, affect deterrence. But the model assumes that a patron and a protégé pay the same amount of peacetime costs and these costs are not negotiable, meaning the politics of the cost-sharing negotiations, as well as its influence on deterrence, is excluded. Johnson (2015) studies endogenous concessions from a protégé and alliance formation, but the challenger does not have the uncertainty over the patron's intervention, which means that the study is silent on the signaling aspect of the cost-sharing negotiations. Fang et al. (2014)'s model also deals with how alliance affects crisis bargaining, but their focus is on how a partner restrains its protégé, and no attention is paid to how the cost-sharing affects the course of crisis bargaining. Iwanami (2023) examines the interaction of burden-sharing and crisis bargaining in a complete-information environment, but the cost transfers between allies and their signaling aspects are not discussed. Johnson and Wolford (2023)'s model investigates the effect of alliances on dispute escalations, but a patron and a protégé do not have negotiations, meaning intra-alliance politics is out of their scope. Fang and Ramsay (2010)'s model studies how NATO countries share a burden to contribute to a public good, but their model does not consider a potential opponent's reaction, and thus, does not take into account its effect on crisis bargaining and its feedback effect on the intra-alliance politics.

5 Analysis

The solution concept is Perfect Bayesian Equilibrium (PBE), in which strategies are sequentially rational and consistent with beliefs updated according to Bayes' Rule whenever possible. First, I focus on the following parameters.

Assumption.

$$p_l > c_T \tag{7}$$

$$\min\{\overline{\beta}(p_m - p_l), \underline{\beta}(p_h - p_l)\} > c_A > \underline{\beta}(p_m - p_l) \tag{8}$$

Line 7 ensures that there is a certain range of x that T prefers war to settlement even if he has to fight alone. In other words, T credibly says that he would rather go to war if he has to give up a lot of the issue, which makes C always think about what range of x is acceptable for T. Line 8 specifies the range of c_A so that A does not always help or always abandon T. Specifically, under this assumption, the high type of A helps T even when they do not have the alliance, but the low type of A helps T if and only if she keeps the alliance. Keeping the alliance brings capability advantages, making the probability of winning high. This high probability of winning incentivizes the low type of A to intervene in war. Hereafter, I call the high type the unconditional type and the low type the conditional type because the intervention by the high type is unconditional on her alliance status, whereas the intervention by the low type is conditional on her alliance status. This assumption means that A's withdrawal from the alliance is dangerous in two ways. First, the lack of the alliance capability advantages emboldens C. Second, the withdrawal induces

uncertainty over A's involvement in a war. Since the unconditional type of A helps T even after withdrawal, but the conditional type does not, C has to guess A's type.

Proposition 1 (Separating 1)**.** *When Assumption is satisfied and*

$$l < 0 \tag{9}$$

$$\overline{\beta} > \frac{\underline{\beta}(p_h - p_l) - c_A}{p_h - p_m - c_T} \tag{10}$$

$$\frac{p_h - p_m - c_T + c_A}{p_h - p_l} > \underline{\beta} \tag{11}$$

$$p_h - p_m > c_T, \tag{12}$$

$$\underline{\beta}(p_h - p_l) < \pi < \overline{\beta}(p_h - p_m - c_T) + c_A, \tag{13}$$

there exists a separating PBE at which the unconditional type of A does not make any cost-sharing demand, C offers $x = p_h - c_T$, and T accepts it, and the conditional type of A offers $a = \min\{a^* = p_h - p_l, \overline{a}\}$, T accepts the offer, C offers $x = p_h - c_T$, and T accepts it on the path of play. See Appendix for proof.

Proposition 1 is a separating equilibrium in which the unconditional A does not make any cost-sharing offers, whereas the conditional A offers $a = \min\{a^*, \overline{a}\}$ and T accepts the offer. The alliance is sustained and war does not happen in both cases. Since the unconditional and conditional A behave differently before crisis bargaining, C (correctly) updates its beliefs. This equilibrium exists when the domestic constituencies are internationalists on average (Line 9), the difference between A's types, and thus the uncertainty of A's types, is large (Line 10 and 11), the capability advantages of the alliance ($p_l - p_m$) is large enough (Line 12), and the total costs of the alliance is neither too small nor too large (Line 13).

The last condition (Line 13) stipulates important off-equilibrium behaviors of A. Due to the relatively large costs of the alliance, the conditional A would withdraw from the alliance if T rejects a cost-sharing offer. In other words, the conditional A abandons the

alliance if the cost-sharing remains the same. Thus, the threat of abandonment is credible for T when A is the conditional type. This off-path behavior incentivizes T to accept the cost-sharing offer in order to keep the alliance and enjoy the capability advantages (Line 12). However, π is so small that the unconditional A would sustain the alliance even when T rejects the offer. So, A cannot credibly threaten that she would abandon T even when the cost-sharing negotiation does not succeed. Because of this, T will be better off rejecting the offer since he can enjoy the benefit of the alliance without paying any extra costs. On such an occasion, the unconditional A cannot get anything from the cost-sharing negotiation, and thus, does not demand anything from the beginning to avoid domestic punishment by internationalists as shown in Line 9.

This equilibrium suggests several significant implications for cost-sharing between allies. First, a credible threat of abandonment is key for successful cost-sharing renegotiations. As discussed above, the incentive to accept a cost-sharing offer emerges when T faces a real threat of abandonment, and thus, T needs to lower the alliance costs to sustain A's involvement. However, T does not agree to pay if A would not leave the alliance, as in the case of the unconditional A. A credible threat of abandonment is necessary for success in cost-sharing. This leads to the next implication.

Second, a successful cost-sharing negotiation is a sign that A is not strongly committed to the alliance, but it also makes A's future support credible. T agrees on the cost-sharing offer only when A is the conditional type and unwilling to keep the alliance if the cost-sharing does not change. The success of the negotiation, thus, updates C's beliefs. C thinks that A that achieves successful cost-sharing is the conditional type and thus less committed to the alliance. In other words, T's acceptance of a cost-sharing offer reveals his doubt in A's

support. T agrees to pay despite its negative impact on signaling because it rather renews the credibility of A's future support by making the alliance reasonable and by maintaining the capability advantages of the alliance (Line 12). Cost-sharing makes the alliance cheaper for A, and she can find the sustainment of the alliance beneficial. This brings capability advantages, leading to (the continuation of) strong deterrence. Put differently, successful cost-sharing signals that A's support is conditional, but simultaneously, it also satisfies the condition for future support by reducing the alliance costs for A. In this sense, successful cost-sharing makes A's commitment credible again. For T, this outcome is better than mimicking T that has the unconditional type of A by rejecting the cost-sharing offer because it would end up letting A leave the alliance due to the high alliance costs (Line 13). This is the reason for T's acceptance of the cost-sharing offer. In more general words, this equilibrium suggests that allies sometimes keep their alliance even though it damages the signaling aspect of the alliance. A protégé prolongs the life of the alliance by making the alliance cheaper for its patron, even though it signals a weaker commitment of the patron. The reason is that the allies can enjoy a good share of the issue and keep high deterrence ($x = p_h - c_T$) as long as they keep the capability advantages of the alliance. This means that allies prioritize the capability benefits over the signaling aspect of the alliance. The literature often emphasizes the signaling aspect of alliance (Morrow, 1994, 2000; Smith, 1995; Kenwick and McManus, 2021), but Proposition 1 implies that alliances sometimes abandon that aspect, but the alliance can still survive and deter an opponent.

Third, cost-sharing is an exchange between money and power. In the equilibrium, T pays the smaller amount between $\overline{a}$ (the maximum amount of its ability to pay) and $a^* = p_h - p_l$. When T's ability to pay is high enough ($\overline{a} > a^*$), he has to pay a^*, and this

is exactly the difference in the value brought by sustaining A's participation through the alliance. Let's take a look at T's off-equilibrium-path behavior. If T did not accept the cost-sharing offer, A would withdraw from the alliance. Then, the share of the issue in crisis bargaining for T would be $x = p_l - c_T$ since C demands a lot and T has to accept it due to the lack of the alliance. The share of the issue on the equilibrium, however, is $x = p_h - c_T$ thanks to the alliance. The improvement of the issue at stake brought by the alliance for T is $(p_h - c_T) - (p_l - c_T) = p_h - p_l$, which is the same as a^*. T buys the enhanced power and enjoys a better share of the issue by paying the same value of money. This result is consistent with Morrow (1991), which argues that allies pay autonomy costs to enjoy the benefit of alliance in asymmetric alliances. But, unlike Morrow (1991), this equilibrium specifies what "autonomy costs" are and shows how the exchange rate is determined. In a practical sense, this implies that the larger power an alliance brings, the more money a protégé has to pay. In fact, Japan and South Korea have many US bases and soldiers in their countries and pay a lot to the US, whereas the Philippines does not have such large US bases and has not paid similar costs in history.

The last feature of this equilibrium is a discrepancy between an ostensibly good division of the issue and the actual payoff for T. As discussed in the previous paragraph, the division of the issue on equilibrium when A is conditional is $x = p_h - c_T$, which is the same as the division when A is unconditional. However, since T burdens some of the costs of the alliance, the final payoff for T when its ability to pay is large enough is $p_h - c_T - a^* = p_l - c_T$, which is indifferent from the payoff when T rejects the cost-sharing offer and loses the alliance. Thus, this equilibrium is featured with (a) an ostensibly good division of the issue and (b) lower payoffs for T. To wit, although the model does not talk about T's

domestic politics, left-wing parties in Japan have strongly criticized the cost-sharing by their government. This may be because of the lack of diplomatic sense or a partisan attitude nested in the Cold War structure, but this surely suggests that elements of the people think that the payment is too expensive and unreasonable. Socialist and communist parties in Japan even claim the abolishment of the US-Japan alliance in history, which is consistent with the implication that the utility of the lack of the alliance is almost the same as the utility of the existence of the alliance with cost-sharing.

Proposition 2 (Separating 2)**.** *When Assumption, Line 10, 11, 12, and 13 are satisfied and*

$$\overline{a} > a^* \tag{14}$$

$$l^* = \frac{(1+\underline{\beta})a^* - \pi}{\overline{a} - a^*} > l \geq 0 \tag{15}$$

there exists a separating PBE at which the unconditional type of A offers $a = \overline{a}$, T rejects it, A does not withdraw from the alliance, C offers $x = p_h - c_T$, and T accepts it, and the conditional A offers $a = a^*$, T accepts the offer, C offers $x = p_h - c_T$, and T accepts it on the path of play. See Appendix for proof.

Proposition 2 is a separating equilibrium where the unconditional A offers $a = \overline{a}$, T rejects it, and A does not withdraw from the alliance. Here, the cost-sharing negotiation fails but the alliance is sustained. The conditional A, however, offers $a = a^*$, T accepts it, and thus, the cost-sharing negotiation is successful and they keep the alliance. C correctly updates its belief since the course of the cost-sharing negotiation is different depending on the types of A. Proposition 2 exists under similar parameters of Proposition 1. It exists when the uncertainty is large (Line 10 and 11), the capability advantages of the alliance are large enough (Line 12), and π is not too large but not too small (Line 13). The key difference from Proposition 1 is the equilibrium behavior for the unconditional A, which is

driven by the strong but not too strong domestic isolationism in A (See Line 15).

When A is unconditional, it offers the maximum level of cost-sharing ($a = \overline{a}$). Since the unconditional A highly evaluates the issue at stake, she cannot make a credible threat of withdrawal. Knowing this, T does not agree to pay even a very small amount of the cost-sharing demand, and the cost-sharing negotiation is doomed to fail. Thus, the unconditional A cannot get anything directly from the negotiation due to her strong commitment. On that occasion, A maximizes her payoffs by offering the maximum level of cost-sharing demand and pandering to her domestic isolationism (See $l > 0$ in Line 15)[11]. Note that, in previous Proposition 1, the unconditional A makes "no demand" because of the internationalism in her country (Line9). This is not the case in Proposition 2. This is the main difference from the previous equilibrium.

Proposition 2 is interesting because it shows a *positive* correlation between the resolve of A and the cost-sharing demand. When A's value of the issue is high and thus A is unconditional ($\overline{\beta}$), she offers $a = \overline{a}$. When A's value of the issue is low and thus A is conditional ($\underline{\beta}$), she offers $a = a^*$, which is strictly lower than $\overline{a}$ (Line 14). In other words, the more committed A is, the more she demands. This *positive* correlation occurs in equilibrium due to the combination of domestic isolationism and the lack of credible threat of abandonment. As explained above, A's unconditional help makes the threat of abandonment not credible for T, which leads to no concession by T in the cost-sharing negotiation. Given this, A demands as much as possible to pander to the domestic isolationists, knowing that the negotiation would probably not succeed. The conditional A,

[11] We could assume a different (or more general) function that maps a cost-sharing offer to benefit from domestic politics, but this would not change the interpretation of the results: When the domestic constituencies are isolationists, the unconditional A demands a large and optimal cost-sharing offer and pander to the domestic constituencies knowing that the demand will be rejected by T.

however, does not demand a lot despite the same level of domestic isolationism because she can get a tangible benefit from the negotiation. Such A offers a lower cost-sharing demand than the unconditional A and gets the benefits from both the cost-sharing negotiation and partially, if not completely, satisfied domestic constituencies. This is the reason for a large demand for the unconditional type and a lower demand for the conditional type.

An alternative interpretation of this result is that domestic isolationism can appear in the cost-sharing negotiation only when the patron's commitment is strong. When the patron is the unconditional type and her domestic constituencies are isolationists, she offers the same cost-sharing demand as a patron that has domestic internationalism ($a = a^*$. See Proposition 1), implying an inability to identify the domestic situations based on international negotiation when the patron is the conditional type. On the contrary, when a patron is the unconditional type, the cost-sharing demand gets large when domestic constituencies are isolationists ($a = \overline{a}$), whereas domestic internationalism eliminates such negotiations from the empirical record because a patron does not offer anything in the first place (See Proposition 1). This shows how domestic politics can be reflected in international cost-sharing negotiations when a patron's commitment is strong, not when it is weak.

Proposition 2 also explains why a large cost-sharing demand has a deterrence effect, even though such a demand makes the cost-sharing negotiation fail. When T rejects a large cost-sharing offer and the negotiation fails, C infers that this is a sign of T's confidence in A's support. In other words, when C observes that A could not achieve a concession from T, it thinks that T rejected the cost-sharing offer because A would not withdraw from the alliance. And because A would not withdraw from the alliance, C reasons that she must

be the unconditional type. On such an occasion, the large cost-sharing offer makes sense for C given A's commonly known high domestic isolationism: it understands that a large cost-sharing demand is driven by the domestic isolationism. As such, the negotiation fails due to A's large offer, but this failure signals A's strong commitment and panders to the domestic isolationism at the same time. This is the second way of achieving peace through cost-sharing negotiations.

This result may sound counterintuitive, but this is still within the conventional costly-signaling mechanism (Fearon, 1997). The failure of cost-sharing negotiations happens when A is the unconditional type, meaning she is (not happy but) fine with *continuing* to pay the full costs of the alliance, whereas the success of the negotiation means that the alliance becomes *less* costly for A. Thus, the failure of the negotiation is more costly for A than the success of the negotiation, and thus, the failure ends up revealing the high resolve of A. This result is consistent with recent research arguing that free-riding by a protégé on a patron's military capability rather signals the patron's high resolve (Smith and Dong, 2024)[12].

In reality, when people see a large cost-sharing demand, they worry that it is a harbinger of fragile alliances and an unstable world. This worry is probably valid in that it is a sign of strong domestic isolationism, which could lead to disagreements between allies and perhaps the termination of an alliance in the long run (see the next proposition for more on this). But Proposition 2 shows that a large cost-sharing demand panders to domestic isolationism, which may open up a way to prolong the life of the alliance by satisfying the

[12] Becker et al. (2023) recently reported that US president's negative rhetoric does not increase (rather decreases) NOTO countries' military contribution. Proposition 2 and Smith and Dong (2024) answer why the US allows such free-riding and why it has a positive influence on deterrence.

domestic constituency. Also, the model shows that a cost-sharing demand is *amplified due to the strong commitment of the patron and the protégé's confidence in its support*. If a patron's intervention is conditional, the cost-sharing negotiation succeeds with a relatively smaller demand, since a patron tries to win both international concession and domestic support and thus has an incentive to lower the demand to make it easier for its protégé to accept the demand. When the patron is unconditional, however, this incentive does not work. The protégé loses the incentive to accept even a very small demand, and this rather increases the patron's demand to secure the domestic benefit.

Proposition 3 (Separating 3)**.** *When Assumption, Line 10, 11, 12, and 13 are satisfied and*

$$1 > l > l^* \tag{16}$$

there exists a separating PBE at which the unconditional type of A offers $a = \overline{a}$, T rejects it, A does not withdraw from the alliance, C offers $x = p_h - c_T$, and T accepts it, and the conditional A offers $a = \overline{a}$, T rejects the offer, A withdraw from the alliance, C offers $x = p_l - c_T$, and T accepts it on the path of play. See Appendix for proof.

Proposition 3 is a separating equilibrium where both the unconditional and conditional types of A demand the highest cost-sharing, $a = \overline{a}$, and the offer is rejected in both cases. The separation happens after the failure of the cost-sharing renegotiation. The unconditional A remains in the alliance and obtains a better division of the issue ($x = p_h - c_t$), whereas the conditional A withdraws from the alliance and secures a very minimum division of the issue ($x = p_l - c_T$).

Proposition 3 is an equilibrium of alliance termination due to extremely high isolationism. The strategy for the unconditional A is the same as Proposition 2: her strong commitment fails the cost-sharing renegotiation in any case, so she panders to domestic

isolationism and does not withdraw from the alliance after the failure of the negotiation. But the conditional type of A behaves differently from Proposition 2. As Line 16 suggests, her extremely strong domestic isolationism makes an offer very demanding, even though it could achieve a successful cost-sharing negotiation with lower demand, as in Proposition 2. This is because the benefit from extreme domestic isolationism dominates the benefit from successful negotiation. By offering $a = \overline{a}$, the conditional type gets $l\overline{a}$, and since the value of l is high, the benefit from domestic politics is large. This large domestic benefit justifies the deficit of the alliance termination and the smaller division of the issue. This equilibrium shows that, although the unconditional type can sustain the alliance even after the negotiation failure, the cost-sharing negotiation cannot save the alliance when the patron's value of the issue is not high ($\underline{\beta}$) and domestic isolationism is extremely strong.

This equilibrium partially confirms our intuition: a very high cost-sharing demand can be a terminal diagnosis for alliances. Such demand eliminates the negotiable range for allies, and their alliance will end. Coupled with Proposition 2, however, this equilibrium also shows that a very high cost-sharing demand is not a sufficient condition for alliance termination. As is the case in the unconditional type of A in Proposition 2 and 3, a patron raises a cost-sharing demand to get a domestic win, not an international win. The model suggests that it is difficult to diagnose the future of an alliance just with an extreme cost-sharing offer. A careful analysis of domestic politics (whether l is higher than l^*) and the value of the issue for A (whether β is low ($\underline{\beta}$)) are important to disentangle these cases.

Proposition 4 (Pooling 1)**.** *When Assumption, Line 10, 11, and 12 are satisfied and*

$$l < 0 \tag{17}$$

$$\pi < \underline{\beta}(p_h - p_l) \tag{18}$$

$$d < d^* = \frac{p_m - p_l}{p_m - p_l + c_T + c_C}, \tag{19}$$

where d is C's belief that A is the unconditional type after withdrawal, there exists a pooling PBE at which both types of A do not make a cost-sharing offer, C offers $p_h - c_T$, and T accepts it regardless of A's type on the path of play. See Appendix for proof.

Proposition 5 (Pooling 2)**.** *When Assumption, Lines 10, 11, 12, 18, and 19 are satisfied and*

$$l > 0 \tag{20}$$

there exists a pooling PBE at which both types of A offer $a = \overline{a}$, T rejects regardless of A's type, both types of A do not withdraw from the alliance, C offers $p_h - c_T$, and T accepts it regardless of A's type. See Appendix for proof.

Proposition 4 and 5 show two similar pooling equilibria where both types of A sustain the alliance due to a very small value of π (Line 18)[13]. Because of the small costs of the alliance, both types of A cannot make a credible threat of abandonment, and T does not accept any cost-sharing offer. Given this, both types of A make "no demand" to avoid domestic punishment when her constituencies are internationalists in Proposition 4 (See Line 17) or make the largest demand to benefit from domestic rewards when they are isolationists in Proposition 5 (See Line 20). Proposition 4 is what people usually think of as a normal alliance: The alliance continues to exist without any conflicts over cost-sharing. Proposition 5 is an equilibrium of uninformative cost-sharing demands. Unlike

[13] As Line 19 shows, A's strategy is also supported by C's off-equilibrium-path belief that the withdrawal from the alliance is a sign of the conditional A. These two pooling equilibria pass the Intuitive Criterion (Cho and Kreps, 1987). See Appendix for more details.

Proposition 2, a high cost-sharing demand does *not* update C's belief due to the identical pre-crisis-bargaining behavior of A.

6 Empirical Records

6.1 Japan in 1978

In 1978, Japan started shouldering the costs of its alliance with the US outside of the original framework of the alliance treaty. In the 1960 agreement between Japan and the US (Japan Status of Forces Agreement (SOFA)), which is still effective in 2024, it is stipulated that the US should "bear...all expenditures incident to the maintenance of the United States armed forces in Japan" except providing lands, areas, and rights of way.[14] Nonetheless, Japan started paying some of the costs of the US bases in Japan outside of the agreement, including salaries of workers in US bases, utility bills, and construction costs for US facilities such as houses and amusement facilities. Since then, Japan's cost-sharing has increased over the years, and Japan's cost-offset percentage reached 74.5% of the total costs of US bases in Japan in 2002 according to the Department of Defense's report.[15]

I argue that 1978 was a key moment for the cost-sharing of Japan and that Japan entered the first separating equilibrium (Proposition 1) from the first pooling equilibrium (Proposition 4) in 1978. Following the advice by Goemans and Spaniel (2016), I offer three sets of evidence. In the model, actors' behavior on equilibrium moves from Proposition 4 to Proposition 1 when (a) the cost of the alliance, π, becomes higher for A and (b) the

[14] Article XXIV of the Status of Forces Agreement (SOFA) between the United States and Japan

[15] the Department of Defense, USA. "2004 Statistical Compendium on Allied Contributions to the Common Defense". (URL: https://apps.dtic.mil/sti/citations/ADA475431. Last Access: August 17, 2024)

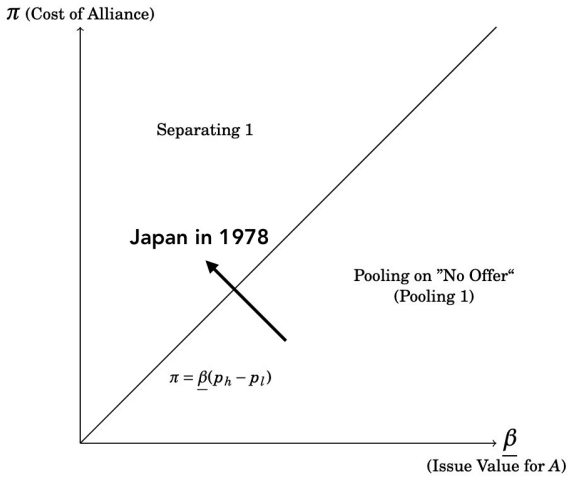

Figure 2: Key Equilibria of the Model

value of the issue becomes lower for the conditional A (β. See Figure 2 for graphical understanding[16]), which creates larger uncertainty over A's commitment (Line10). Also, one key strategic consideration in the separating equilibrium is that (c) T accepts the cost-sharing offer because it worries that A will withdraw from the alliance otherwise. I will show that the costs of the alliance for the US was dramatically increasing, that the value of the alliance was getting lower at that time, which created larger uncertainty of the US commitment, and that the US administration had a strategy to threaten to withdraw from the alliance, and such a threat was probably credible for Japan.

First, I provide evidence that the cost of the alliance was getting higher for the US. Japan's economic growth made the US bases in Japan more expensive due to weaker US dollars, inflation in Japan, and an increase in salaries for Japanese workers. The USD/JPY exchange rate surged from 1976 to 1978. In 1976, one dollar was 296 yen, but it became

[16]Note that the threshold between Proposition 1 and Proposition 4 is $\pi = \underline{\beta}(p_h - p_l)$ as shown in Line 10 and 18. This is a clear cut, meaning that actors *must* enter Proposition 1 (or 2) as β decreases and π increases before they enter other types of equilibrium.

206 yen in 1978. This means the value of US dollars against Japanese yen has decreased by two-thirds in two years. In addition, inflation in Japan made it more expensive to sustain the US bases in Japan. The Oil Shock caused inflation, and the inflation rate from 1975 to 1978 in Japan was 23%. The minimum wage in Okinawa, where the majority of US bases are located, increased by 7% only in 1978. These factors increased the costs of US bases in Japan.[17]

Key people in the US and Japanese governments stated the impact of the increasing costs of US bases in Japan. In the Japanese diplomatic archive on a Japan-US meeting on March 21, 1977, President Carter was reported to state that the personal costs (salaries for workers) of the US forces stationed in Japan increased and this was a problem in the US Congress, so he would appreciate it if the prime minister of Japan considers Japan's cost-sharing.[18] Shin Kanemaru, who took the initiative in the new decision of the Japanese government as a Director General of the Japan Defense Agency, recalled his memory that on April 4, 1978, Commander George Loving came and said that strong Yen and weak Dollars were causing trouble and he wanted Shin Kanemaru to do something (Kanemaru, 1979, pp. 78-79). All of these things show that the costs of the US-Japan alliance were getting expensive for the US, suggesting a higher value in π.

Second, the value of the issue that the alliance protects, β, decreased in 1978. The 1970s were an era of US withdrawal from Asia, evidenced by the improved US-China relationship, the bitter defeat in Vietnam, and a series of withdrawal plans from Asian countries, such as Thailand, Taiwan, and South Korea. The Nixon Doctrine in 1969

[17] In addition, the inflation in the late 1970s in the US would have increased the domestic pressure to cut down the expenses of US bases overseas.

[18] Telegram from U.S.A. to Tokyo, No.1286 (March 22, 1977), 2014-4987, Diplomatic Archives of the Ministry of Foreign Affairs Japan

stipulated the US grand strategy in the next decade. After Nixon visited China in 1972, it was evident that the US was seeking to improve its relationship with China. Under the Carter administration, the US-China (PRC) relationship became normalized and they announced that they would establish a formal diplomatic relationship in December 1978[19]. As a result, the US terminated its alliance with Taiwan[20] and withdrew its military forces from the country in 1979. China is not the only example. In 1975, the Vietnam War ended, in which US bases in Japan were used as bases for fighters and other aircraft. This decreased the necessity of US bases in Japan. The end of the Vietnam War pushed the US to withdraw its military forces from Thailand in 1976. In addition, Carter won the presidential election in 1976 with a promise that the US would withdraw from South Korea. This plan was not carried out in the end, but, coupled with the US plan to pivot to NATO, it raised a question of US commitment to East Asia (Green, 2017, p. 377), suggesting larger uncertainty over the US's intentions. The Japanese side recognized the US's reluctant attitude. In a section titled "Does the US not help Japan any longer?" in his autobiography, Shin Kanemaru wrote that the structure of the US-Japan alliance was changing and that, with the withdrawal of the US forces from East Asia, "we can not deny that the importance of military intervention (for the US) will be decreasing in these regions"(Kanemaru, 1979, p. 76). All of these things suggest that Japan was concerned that the US's benefits of sustaining US bases in East Asia were decreasing due to the disappearance of China and Vietnam as enemies, which were clarified by a series of withdrawal plans from Asian

[19]Website of Office of the Historian, *China Policy* (URL: https://history.state.gov/milestones/1977-1980/china-policy. Last Access: August 16th, 2024)

[20]The full name of the treaty is the Mutual Defense Treaty between the United States and the Republic of China, which was signed in 1954. This treaty requires Taiwan and the US to protect each other when either of them is attacked, but the regional application of the treaty is limited to mainland Taiwan and the Pescadores, whereas Kinmen and Matsu islands were not included.

countries. This implies that the value of the issue at stake ($\underline{\beta}$) for the US was decreasing in 1978 and that this created larger uncertainty of the US's commitment, satisfying an important condition of Proposition 1 (Line 10 and 11).

Finally, an important strategic interaction in Proposition 4 is that the threat of withdrawal is credible. Although the US tried to convince that the US would not abandon Japan, and the Japanese government acknowledged that point in official meetings[21], key people on the Japanese side were still skeptical. A Japanese historian points out that reduction plans of US Marines in Okinawa and their new rotation routine caused concern about the US's future presence in Japan (Nozoe, 2014). People at the Japan Defense Agency were reported to say that they needed to think that the US land power would disappear in the Far East in the 1980s and that the withdrawal of the US Marines in Okinawa was a matter of time (Nozoe, 2014). Green (2017, p. 380) describes the US-Japan relationship at that time as follows. "(T)he U.S.-Japan alliance had been drifting since the Nixon shocks, and there was growing evidence that Tokyo might be hedging for the possibility of a post-American Pacific order", suggesting that Japan was preparing for the US's additional withdrawal from the region, where Japan was one of a few countries that remained.

In fact, there is suggesting evidence that the US government exploited Japan's worry about the US commitment to achieve concessions from the Japanese side. On June 19, 1978, Mike Armacost, a member of the National Security Council in the Carter administration, gave pieces of advice to Zbigniew Brzezinski, an Assistant to the President for National Security Affairs, about his upcoming meeting with Shin Kanemaru. In the memorandum, Mike Armacost stated, "(Y)ou should...point out the impact of escalating

[21] This must have contributed to the larger uncertainty as well. (Line 10 and 11)

costs on the willingness of the services to maintain forces in Japan" and "that a sizable GOJ cost-sharing package would have enormous value in solidifying our security and political relationship [22]." This clearly shows that the US had a strategy to achieve a concession from Japan by threatening that Japan's cost-sharing would affect the continuation of US bases in Japan. Coupled with the US withdrawal from other Asian countries and Japan's worry that the same thing would happen in Japan soon, this threat must have been credible for Japan. This is exactly what the model predicts as conditions of successful cost-sharing.

6.2 Japan in 2019

Japan in 2019 is another example of cost-sharing negotiations between allies. It is famous that Donald Trump was not satisfied with Japan's cost-sharing for the US-Japan alliance, even before his presidency (Sanger and Haberman, 2016). In November 2019, it was reported that the Trump administration asked Tokyo to pay four times as much to offset the costs of US forces in Japan (Seligman and Gramer, 2019), which would be $8 billion according to John Bolton, former National Security Advisor (Bolton, 2024, Ch.11). The cost-sharing agreement effective at that time would expire in March 2021. So, Japan and the US officially started the negotiation around October 2020 to discuss the next 5-year cost-sharing, although the substantive negotiations between practitioners arguably began by July 2019 (Bolton, 2024, Ch.11). However, after a series of negotiations, time ran out, and it was announced that the Trump administration and Japan gave up reaching an agreement (Tajima, 2020). Why did the US and Japan fail to reach a cost-sharing agreement?

[22] Memorandum 3766 from Mike Marmacost to Zbigniew, Brzezinski's Country Files, Records of the Office of the National Security Advisor (Carter Administration), Box 40, Japan, 6-12/78. Jimmy Carter Library.

Although details about the negotiation have not been disclosed yet, Proposition 2 provides an important interpretation of the failure of the cost-sharing negotiation between the Trump administration and Japan. Proposition 2 tells us that the failure of a cost-sharing negotiation happens when a patron is unconditional ($\overline{\beta}$), the threat of abandonment is not credible, and the domestic constituencies are isolationists but not too much, as Line 15 requires. In such a situation, the proposition says that a patron mainly tries to satisfy her domestic isolationism, which leads to a very high cost-sharing demand ($a = \overline{a}$). To prove the usefulness of the model, I empirically indicate that actors' important strategic interactions in Proposition 2 actually happened in Japan in 2019. Specifically, I show that (a) Trump tried to use a threat of withdrawal to achieve a concession, (b) this threat was probably not credible for Japan because of the strong commitment ($\overline{\beta}$), and (c) Trump was mainly concerned with domestic politics, where isolationism was strong enough ($l > 0$).

First, Trump was aware of the threat of withdrawal as a strategy to induce a concession from Japan. In his tell-all book, Bolton writes that "(H)e (Trump) said, as he did more and more frequently, that the way to get the $8 and $5 billion annual payments, respectively (Japan and South Korea), was to threaten to withdraw all US forces" (Bolton, 2024, Ch.11). Trump added, "That puts you in a very strong bargaining position" (Bolton, 2024, Ch.11). This suggests that Trump understood the threat of withdrawal as one of his cards at hand, which is an important moving part of the formal model. There is little doubt that Trump was thinking of the withdrawal as an actual policy option (Jacobs, 2019), but at the same time, he used his position as leverage to achieve a concession.

Second, the threat of abandonment was probably not credible for Japan because of the US's strong commitment to Japan's security ($\overline{\beta}$). Whether Trump was serious about

withdrawing from Japan was an important question for the Japanese government. Although Japan was probably not completely sure about what Trump was thinking, there is suggestive evidence that Japan is relatively confident in US support. Japan's optimism came from the good personal relationship between Trump and Shinzo Abe, the Prime Minister of Japan at that time, and the continuation (or possibly improvement) of US security policy for Japan in the Trump administration.

It is well known that Abe and Trump had a honeymoon relationship, as evidenced by the fact that they "met 20 times, played 5 rounds of golf, and had 32 phone calls, at times speaking twice a week" (Teraoka, 2020). Bolton (2024, Ch.11) describes their relationship as follows, "In my view, Trump's best personal relationship among world leaders was with Abe (golf buddies as well as colleagues)." Abe himself recalls his relationship with Trump, "Trump had no problem with calling me for an hour, or an hour and a half when it was long...I tried to honestly tell my thoughts to Trump, and I believe that Trump told me his true thoughts on many issues as well." (Abe et al., 2023, Ch.6)[23]. Abe's investment in the personal relationship with Trump was fruitful to the extent that he successfully "convince(d) the United States to support his strategy in the Indo-Pacific, while drawing attention to issues of importance to Japan" (Teraoka, 2020), implying that the personal relationship was influential enough to pin down Japanese security environment in Trump's mind.

Based on this personal relationship, the Trump administration often confirmed its support for Japan. In February 2017, soon after his inauguration, Trump said after a meeting with Abe, "This administration is committed to bringing those ties even closer.

[23]Translated by the author

We are committed to the security of Japan and all areas under its administrative control and to further strengthening our very crucial alliance" (Hirschfeld and Baker, 2017). This commitment did not change after two years, when Trump became more critical of Japan. In a meeting in Osaka, Japan, "Abe and Trump reconfirmed that the nations' long-standing cooperation on security arrangements remain in place" (Hara, 2019). The continuation of US security policy in the Trump administration may be caused by bureaucratic constraint to some extent (Drezner, 2019), but Trump's strong commitment appears even when he personally criticized Japan: Trump condemned Japan by saying "If Japan is attacked, we will fight World War Three...We will fight at all costs, right? But if we're attacked, Japan doesn't have to help us at all. They can watch it on a Sony television" (Sieg and Leussink, 2019), suggesting that Trump was not satisfied with Japan's role in the alliance for sure, but, even so, this statement is implicitly based on the assumption that the US is ready to protect its ally in the Far East.

The Japanese government's evaluation of the Trump administration can be seen in an article written by an anonymous foreign ministry official. The article admits that "it (Trump's use of economic leverage against its allies) raised doubts in many minds across the region as to the credibility of American security guarantees and commitments" (this suggests the larger uncertainty for a third actor, which makes Line 10 and 11 satisfied more likely), but the article also claims that Japan surely prefers Trump's security polity to Obama's one by saying "do we want...to go back to the world before Trump? For many decision-makers in Tokyo, the answer is probably no" (Y.A., 2020), implying the continuation (or possibly improvement) of the US commitment in Japan in the Trump era. The Japanese people seemed to recognize this point as well, as shown by 82% and 67%

of people expressing their confidence in US support in a war with North Korea and China, respectively, a seven-point increase from the Obama years in the latter number[24].

All of these things suggest Trump's high resolve for Japanese security ($\overline{\beta}$) and Japan's confidence in Trump's support. Proposition 2 tells us that the strong resolve creates an incentive for an ally not to accept any additional cost-sharing since the ally can benefit from the alliance without paying such costs. This is exactly what happened in reality, as shown by the failure of the cost-sharing negotiation in 2019. Moreover, this view provides an answer to the question of why Trump did not reduce his cost-sharing demand to make it easy for Japan to accept. Given that the next presidential election was getting closer and Trump often criticized Japan's free-riding on the alliance, it was a good chance to appeal his competence. The model explains that Japan's confidence in Trump's strong support vanished Japan's incentive to pay *any* extra money, making lowering the cost-sharing demand meaningless.

Finally, Proposition2 states that when the threat of withdrawal is not credible, a patron gives up getting concessions from T and tries to satisfy domestic isolationism with a very high demand. Empirical evidence suggests that domestic isolationism was strong among Trump supporters, and Trump was mainly concerned with domestic politics. Pew Research Center reports that "55% of conservative Republicans say the U.S. should follow its own national interests even when allies disagree" in 2019 (Doherty et al., 2019), which is good evidence that domestic constituencies, especially Trump supporters, are highly isolationists on average at that time ($l > 0$)[25], but not extremely high, as required by Line 15.

[24] See (Green, 2022, p.102) for more on this.

[25] To wit, McDonald et al. (2019) empirically shows that citizens do not care about politicians' consistency as much as they are often assumed and do not punish politicians who change their minds, especially in the Trump era. This suggests that Trump's partisanship makes it difficult to generate audience costs or sunk

There is plenty of evidence that Trump's foreign policy attitude was targeting these domestic supporters. Many scholars would agree that Trump is a populist (see, for example, Boucher and Thies (2019) and Hafner-Burton et al. (2019)), which is characterized by anti-elitism and the general will of the "pure" people. Boucher and Thies (2019) empirically shows that Trump's populist rhetoric created a unique cluster in social media and divided the public and elites on foreign policy issues. Moreover, Bolton said in an interview with a Japanese newspaper as follows, "Trump's yardstick was 'how it affects domestic politics', even in national security issues. If Trump ever decided correctly, this is not because his advisors successfully convinced him but because Trump worried that Republican politicians would oppose otherwise" (Sawamura, 2020)[26]. This clearly states that the main driver of Trump's foreign policy was domestic politics, not international politics.

In such a situation, Proposition 2 predicts that a patron demands the largest amount of cost-sharing, which actually happened in 2019. As explained above, the Trump administration demanded that Japan pay four times as much, which is about $8 billion. This number did not come from a realistic calculation, as shown in Bolton (2024)'s recall that "only Trump knew what payment would satisfy him, so there was no point now trying to guess what the 'real' number was. Trump himself didn't know yet." This is consistent with Proposition 2 in which an unconditional patron demands the largest amount of cost-sharing to satisfy domestic isolationism ($a = \overline{a}$), not to achieve international concessions. The model explains that Trump *had to* primarily concern the domestic politics because his relatively high interest in Japanese security reduced the room for international achievement.

costs, implying that the model setting about domestic politics parameter (l) captures this feature well.

[26]Translated by the author.

7 Conclusion

Japan and South Korea pay a large part of the costs of US bases in their countries, such as salaries to general workers, utilities bills, and construction fees for houses, golf courses, and movie theaters that are exclusively for US soldiers. Such cost-sharing would not increase their military capability, and, according to the alliance literature (Morrow, 1994; Smith, 1995), it should weaken the deterrence by making the alliance cheaper for a patron. These allies, however, often claim a positive effect of alliance cost-sharing on deterrence. Why?

To explain this puzzle, I constructed a formal model in which allies negotiate their cost-sharing under the shadow of international crisis and domestic politics. The model identifies that a credible threat of withdrawal from an alliance is key to successful cost-sharing negotiations. If the threat of abandonment is not credible, a protégé does not have an incentive to accept any cost-sharing.

Moreover, the model shows that both successful and failed cost-sharing negotiations bring peace, but in different ways at different times. First, successful negotiations happen when a patron's threat of withdrawal is credible, and thus, it reveals her weak commitment to the alliance. However, it also sustains the patron's participation in the alliance by reducing the alliance costs, leading to the continued capability advantages of the alliance. The continued capability advantages improve the bargaining power of the allies in crisis bargaining, and thus, bring peace and deterrence. It implies that allies sometimes prioritize the capability advantages of alliances over the signaling aspect and that alliances can survive and deter an opponent even after losing their signaling aspect.

Second, a large cost-sharing demand makes the negotiations fail, but it signals the patron's strong commitment and panders to domestic isolationism at the same time. When

a patron's commitment is strong, a threat of withdrawal is not credible, leading to the rejection by a protégé in the cost-sharing negotiations. This rejection reveals a protégé's confidence in its patron's unconditional support and signals the strong commitment of the patron. When domestic isolationism is strong enough, the patron makes a large cost-sharing demand, even though it will be rejected by her protégé, to maximize her payoff by pandering to her domestic isolationism. This suggests a *positive* association between a patron's strong commitment and a large cost-sharing demand.

Empirical records of cost-sharing negotiations in the US-Japan alliance in 1978 and 2019 show the usefulness of the model. In 1978, the Carter administration succeeded at a cost-sharing negotiation with Japan thanks to its credible threat of withdrawal. In 2019, Trump failed at the cost-sharing negotiations due to the non-credible threat of abandonment, and he maximized his payoffs by pandering to his isolationist supporters. The model provides useful explanations behind the cases.

Overall, this article presents that cost-sharing negotiations work as a regulating valve to regain deterrence both when a patron is committed and not committed. Thus, cost-sharing negotiations allow alliances to survive longer by overcoming various changes in the strategic environment. In a world where intra-alliance disagreement is more and more prevalent, this article provides several useful insights. Moreover, this article calls for more attention to the interdependency among signaling, cost-sharing, and domestic politics in alliance politics.

Some notes on counterfactual scenarios would be useful. In this article, the success and failure of cost-sharing negotiations emerge as a result of strategic interdependence, meaning that changing actors' decisions without such strategic considerations may not work as

intended. For example, rejecting a patron's demand when the patron is not strongly committed would not automatically be costly signaling. Rather, it would trigger the withdrawal from the alliance, causing an unwanted consequence. Similarly, accepting a cost-sharing demand would not help an alliance survive longer when a patron is already committed. It decreases the protégé's payoff for nothing and induces unnecessary uncertainty, leading to the failure of deterrence. This article does not intend to make any specific policy recommendations without careful thought.

References

Abe, S., Hashimoto, G., Oyama, H., and Kitamura, S. (2023). *Abe Shinzo Kaikoroku (Memoir of Shinzo Abe)*. Chuokoron-Shinsha.

Alley, J. and Fuhrmann, M. (2021). Budget breaker? the financial cost of us military alliances. *Security Studies*, 30(5):661–690.

Becker, J., Kreps, S. E., Poast, P., and Terman, R. (2023). Transatlantic shakedown: Presidential shaming and nato burden sharing. *Journal of Conflict Resolution*, page 00220027231167840.

Blankenship, B. (2020). Promises under pressure: Statements of reassurance in US alliances. *International studies quarterly: a publication of the International Studies Association*.

Blankenship, B. (2021). The price of protection: Explaining success and failure of us alliance burden-sharing pressure. *Security Studies*, 30(5):691–724.

Blankenship, B. D. (2023). *The Burden-Sharing Dilemma: Coercive Diplomacy in US Alliance Politics*. Cornell University Press.

Bolton, J. (2024). *The room where it happened: A White House memoir*. Simon and Schuster.

Boucher, J.-C. and Thies, C. G. (2019). "i am a tariff man": The power of populist foreign policy rhetoric under president trump. *The Journal of Politics*, 81(2):712–722.

Cho, I.-K. and Kreps, D. M. (1987). Signaling games and stable equilibria. *The Quarterly Journal of Economics*, 102(2):179–221.

Cooley, A., Nexon, D. H., Poast, P., Alley, J., and Fuhrmann, M. (2022). Estimating alliance costs: An exchange. *Security Studies*, 31(3):510–532.

Doherty, C., Kiley, J., and Asheer, N. (2019). In a politically polarized era, sharp divides in both partisan coalitions. *PEW Research*.

Drezner, D. W. (2019). Present at the destruction: The trump administration and the foreign policy bureaucracy. *The Journal of Politics*, 81(2):723–730.

Fang, S., Johnson, J. C., and Leeds, B. A. (2014). To concede or to resist? the restraining effect of military alliances. *International Organization*, 68(4):775–809.

Fang, S. and Ramsay, K. W. (2010). Outside options and burden sharing in nonbinding alliances. *Political research quarterly*, 63(1):188–202.

Fearon, J. D. (1997). Signaling foreign policy interests: Tying hands versus sinking costs. *Journal of conflict resolution*, 41(1):68–90.

Goemans, H. and Spaniel, W. (2016). Multimethod research: A case for formal theory. *Security Studies*.

Green, M. (2022). *Line of Advantage Japan's Grand Strategy in the Era of Abe Shinzō*. Columbia University Press.

Green, M. J. (2017). *By more than providence: Grand strategy and American power in the Asia Pacific since 1783*. Columbia University Press.

Hafner-Burton, E. M., Narang, N., and Rathbun, B. C. (2019). Introduction: What is populist nationalism and why does it matter? *The Journal of Politics*, 81(2):707–711.

Hara, K. (2019). Japan's abe reaffirms us security ties after trump's verbal swipe. *Nikkei Asia*. Available at:https://asia.nikkei.com/Politics/G-20-summit-Osaka/Japan-s-Abe-reaffirms-US-security-ties-after-Trump-s-verbal-swipe.

Hille, K., White, E., Riordan, P., and Reed, J. (2020). The trump factor: Asian allies question america's reliability. *Financial Times*.

Hirschfeld, J. and Baker, P. (2017). In welcoming shinzo abe, trump affirms us commitment to defending japan. *New York Times*, 20.

Iwanami, Y. (2023). Asymmetric burden-sharing and the restraining and deterrence effects of alliances. *Journal of Peace Research*, page 00223433231158146.

Jacobs, J. (2019). Trump muses privately about ending postwar japan defense pact. *Bloomburg*. Available at:https://www.bloomberg.com/news/articles/2019-06-25/trump-muses-privately-about-ending-postwar-japan-defense-pact.

Johnson, J. C. (2015). The cost of security: Foreign policy concessions and military alliances. *Journal of Peace Research*, 52(5):665–679.

Johnson, J. C. (2022). Designing alliances: How adversaries provoke peacetime military coordination. *International interactions*, 48(1):49–74.

Johnson, J. C. and Leeds, B. A. (2011). Defense pacts: A prescription for peace? *Foreign Policy Analysis*, 7(1):45–65.

Johnson, J. C. and Wolford, S. (2023). Alliance reliability and dispute escalation. *Journal of Conflict Resolution*, 67(4):617–641.

Kanemaru, S. (1979). *Waga Taikenteki Boueiron: Omoiyari No Nichibei Anpo Shin-Jidai [My Experiential Argument for Defense: New Era of US-Japan Security of Sympathy]*. Yell Books.

Kenwick, M. R. and McManus, R. W. (2021). Deterrence theory and alliance politics. In Mitchell, S. M. and Vasquez, J. A., editors, *What do we know about war?*, chapter 3, pages 41–62. Rowman & Littlefield.

Leeds, A. B. and Anac, S. (2005). Alliance institutionalization and alliance performance. *International Interactions*, 31(3):183–202.

Leeds, B., Ritter, J., Mitchell, S., and Long, A. (2002). Alliance treaty obligations and provisions, 1815-1944. *International Interactions*, 28(3):237–260.

Leeds, B. A. (2003a). Alliance reliability in times of war: Explaining state decisions to violate treaties. *International Organization*, 57(4):801–827.

Leeds, B. A. (2003b). Do alliances deter aggression? the influence of military alliances on the initiation of militarized interstate disputes. *American Journal of Political Science*, 47(3):427–439.

Leeds, B. A. and Savun, B. (2007). Terminating alliances: Why do states abrogate agreements? *The Journal of Politics*, 69(4):1118–1132.

McDonald, J., Croco, S. E., and Turitto, C. (2019). Teflon don or politics as usual? an examination of foreign policy flip-flops in the age of trump. *The Journal of Politics*, 81(2):757–766.

Morrow, J. D. (1991). Alliances and asymmetry: An alternative to the capability aggregation model of alliances. *American Journal of Political Science*, pages 904–933.

Morrow, J. D. (1994). Alliances, credibility, and peacetime costs. *The Journal of Conflict Resolution*.

Morrow, J. D. (2000). Alliances: Why write them down? *Annual Review of Political Science*.

Nozoe, F. (2014). "omoiyari yosan" to nichibei kankei 1977-1978 ["sympathy budget" and the us-japan relationship in 1977-1978]. *Okinawa Hougaku*.

Poast, P. (2019). *Arguing about alliances: The art of agreement in military-pact negotiations*. Cornell University Press.

Sanger, D. E. and Haberman, M. (2016). Transcript: Donald trump expounds on his foreign policy views. *New York Times*.

Sawamura, W. (2020). "toranpu-shi, sontoku kanjo ga subete" boruton-shi ni kiku naijitsu ("it's all about profit and loss for mr.trump" asked mr.bolton about inside facts. *The Asahi Shimbun*. Available at:https://www.asahi.com/articles/ASNB14PWQN9ZUHBI012.html?iref=pc_ss_date_article.

Seligman, L. and Gramer, R. (2019). Trump asks tokyo to quadruple payments for us troops in japan. *Foreign Policy*, 15.

Sieg, L. and Leussink, D. (2019). Trump renews criticism of japan-us alliance before g20 summit. *Reuters*. Available at:https://www.reuters.com/article/world/trump-renews-criticism-of-japan-us-alliance-before-g20-summit-idUSKCN1TS051/.

Smith, A. (1995). Alliance formation and war. *International Studies Quarterly*, 39(4):405–425.

Smith, B. and Dong, H. (2024). Free-riding and costly signaling in coalition defenses. *American Political Science Association*.

Smith, B. C. (2021). Military coalitions and the politics of information. *The Journal of Politics*, 83(4):1369–1382.

Tajima, Y. (2020). Japan punts us base funding deal into biden era. *Nikkei Asia*. Available at:https://asia.nikkei.com/Politics/International-relations/Japan-punts-US-base-funding-deal-into-Biden-era.

Teraoka, A. (2020). Resolved: Abe's investment in his relationship with president trump has advanced japanese interests. *Debating Japan*, 3(1).

Tomz, M. and Weeks, J. L. (2021). Military alliances and public support for war. *International Studies Quarterly*, 65(3):811–824.

Tomz, M., Weeks, J. L. P., and Bansak, K. (2023). How membership in the north atlantic treaty organization transforms public support for war. *PNAS nexus*, 2(7):gad206.

Tomz, M., Weeks, J. L. P., and Yarhi-Milo, K. (2020). Public opinion and decisions about military force in democracies. *International organization*, 74(1):119–143.

Wolford, S. (2007). The turnover trap: New leaders, reputation, and international conflict. *American Journal of Political Science*, 51(4):772–788.

Y.A. (2020). The virtues of a confrontational china strategy. *The American Interest*. Available at:https://www.the-american-interest.com/2020/04/10/the-virtues-of-a-confrontational-china-strategy/.

Appendix

Proof of Proposition 1

First of all, I analyze the crisis-bargaining stage by using backward induction. A helps T in a war iff

$$\begin{aligned} & u_A(\text{help}) > u_A(\neg\text{help}) \\ \Leftrightarrow\ & \beta(p_h - c_A) + \lambda_{A,n} > \beta(p_l - c_A) + \lambda_{A,n} \\ \Leftrightarrow\ & \beta(p_h - p_l) > c_A, \end{aligned}$$

when A has an alliance.

A helps iff

$$\begin{aligned} & \beta(p_m - c_A) + \lambda_{A,n} > \beta(p_l - c_A) + \lambda_{A,n} \\ \Leftrightarrow\ & \beta(p_m - p_l) > c_A, \end{aligned}$$

when A does not have an alliance. Thus, under the following condition, the high type of A helps T regardless of her alliance status and the low type of A helps T only when she has an alliance.

$$\min\{\overline{\beta}(p_m - p_l), \underline{\beta}(p_h - p_l)\} > c_A > \underline{\beta}(p_m - p_l)$$

, which is Line 8 in Assumption.

Given this incentive of A, T accepts C's offer, x, in crisis bargaining when A is the unconditional type iff

$$x \geq p_h - c_T$$

T accepts x when A is the conditional type and does not have an alliance iff

$$x \geq p_h - c_T$$

T accepts x when A is the conditional type and does not have an alliance iff

$$x \geq p_l - c_T$$

To make sure that there is a range of $x > 0$, we need to assume

$$p_l > c_T$$

, which is Line 7 in Assumption.

Because of the inefficiency of war, C's offer of x is $x = p_h - c_T$ when A and T have an alliance. When they do not have an alliance, C offers $x = p_m - c_T$ if C is sure that A is the unconditional type and $x = p_l - c_T$ if C is sure that A is the conditional type.

Next, I move on to the analysis in the cost-sharing negotiation stage. In Proposition 1, the unconditional A chooses "no demand" and, in the off-equilibrium path, she stays in the alliance after T's rejection of a. The conditional type of A chooses $a = \min\{a^*, \overline{a}\}$ and, in the off-equilibrium path, she withdraws from the alliance after T's rejection of a. T with the unconditional A rejects any cost-sharing offer, whereas T with the conditional A accepts $a \leq \min\{a^*, \overline{a}\}$.

With the equilibrium behavior of actors in Proposition 2 in mind, I check under what conditions actors do not have profitable deviation.

I first analyze A's decision to remain in or withdraw from the alliance after T's rejection of the cost-sharing offer.

If A remains in the alliance, C's belief is that A is the unconditional type. It offers $x = p_h - c_T$ and T accepts. A pays the entire costs of the alliance. Thus,

$$u_A(\text{stay}) = \beta(p_h - c_T) - \pi + la$$

If A withdraws from the alliance, C thinks A is conditional and offers $x = p_l - c_T$. T with the unconditional A rejects the offer and T with the conditional A accepts the offer.

$$u_A(\text{withdraw}) = \begin{cases} \overline{\beta} p_m - c_A + la & (\text{When } A \text{ is committed}) \\ \underline{\beta}(p_l - c_T) + la & (\text{When } A \text{ is not committed}) \end{cases}$$

When A is unconditional, A remains in the alliance iff

$$\begin{aligned} &\beta(p_h - c_T) - \pi + la > \overline{\beta} p_m - c_A + la \\ &\Leftrightarrow \overline{\beta}(p_h - p_m - c_T) + c_A > \pi \end{aligned} \tag{21}$$

When A is conditional, A withdraws from the alliance iff

$$\begin{aligned} &\underline{\beta}(p_h - c_T) - \pi + la > \underline{\beta}(p_l - c_T) + la \\ &\Leftrightarrow \pi > \underline{\beta}(p_h - p_l) \end{aligned} \tag{22}$$

Next, I analyze T's incentive to accept the cost-sharing offer.

When T's partner A is unconditional,

$$\begin{aligned} u_T(\text{accept}) &= p_h - c_T - a \\ u_T(\text{reject}) &= p_h - c_T \end{aligned}$$

T does never accept the offer since $a > 0$.

When T's partner A is conditional,

$$\begin{aligned} u_T(\text{accept}) &= p_h - c_T - a \\ u_T(\text{reject}) &= p_l - c_T \end{aligned}$$

T accepts a iff

$$\begin{aligned} &u_T(\text{accept}) > u_T(\text{reject}) \\ &\Leftrightarrow a^* = p_h - p_l \geq a \end{aligned} \tag{23}$$

Finally, I analyze A's optimal cost-sharing offer.

First, I consider the unconditional A. When A is unconditional, any offer of cost-sharing is rejected. Thus,

$$\begin{aligned} u_A(\text{no demand}) &= \overline{\beta}(p_h - c_T) - \pi \\ u_A(\text{offer } a) &= \overline{\beta}(p_h - c_T) - \pi + la \end{aligned}$$

Given that $\overline{\beta}(p_h - c_T) - \pi$ is the same in both payoffs, la determines A's action. A makes "no demand" when

$$l < 0 \tag{24}$$

A offers $a = \overline{a}$ when

$$l > 0 \tag{25}$$

because A gets the largest payoff at the largest a.

Second, I consider the conditional A. When A is conditional and T's maximum ability to pay is larger than a^* ($\overline{a} > a^*$), A's payoffs are summarized as follows.

$$\begin{aligned}
u_A(\text{no demand}) &= \underline{\beta}(p_h - c_T) - \pi \\
\max_a u_A(\text{offer } a \le a^*) &= u_A(\text{offer } a = a^*) \\
&= \underline{\beta}(p_h - c_T) - \pi + a^* + la^* \ (\because -1 < l) \\
\max_a u_A(\text{offer } a > a^*) &= \max_a(\underline{\beta}(p_l - c_T) + la) \\
&= \begin{cases} u_A(\text{offer } a = a^* + \epsilon) \approx \underline{\beta}(p_l - c_T) + la^* \text{ when } l < 0 \\ u_A(\text{offer } a = \overline{a}) = \underline{\beta}(p_l - c_T) + l\overline{a} \text{ when } l > 0 \end{cases}
\end{aligned}$$

, where ϵ is a positive and tiny value.

A offers $a = a^*$ when

$$\begin{aligned}
& u_A(\text{offer } a = a^*) > u_A(\text{no demand}) \\
\Leftrightarrow\ & a^*(l+1) > 0
\end{aligned}$$

, which is always true since $l > -1$, and

$$\begin{aligned}
& u_A(\text{offer } a = a^*) > u_A(\text{offer } a > a^*) \\
\Leftrightarrow\ & \begin{cases} (\underline{\beta}+1)a^* > \pi \text{ when } l < 0 \\ \dfrac{(\underline{\beta}+1)a^* - \pi}{\overline{a} - a^*} > l \text{ when } l > 0 \end{cases}
\end{aligned}$$

Thus, when $l < 0$, the conditional A offers a^* if

$$(\underline{\beta}+1)a^* > \pi \tag{26}$$

and when $l > 0$, the conditional A offers a^* if

$$l^* = \frac{(\underline{\beta}+1)a^* - \pi}{\overline{a} - a^*} > l \tag{27}$$

When A is conditional and T's maximum ability to pay is smaller than a^* ($\overline{a} < a^*$), any offer of cost-sharing is accepted by T. Thus, A's payoffs are expressed as follows.

$$\begin{aligned}
u_A(\text{no demand}) &= \underline{\beta}(p_h - c_T) - \pi \\
\max_a u_A(\text{offer } a) &= \max_a(\underline{\beta}(p_h - c_T) - \pi + a + la) \\
&= u_A(\text{offer } \overline{a}) \ (\because -1 < l) \\
&= \underline{\beta}(p_h - c_T) - \pi + \overline{a} + l\overline{a}
\end{aligned}$$

By comparing these payoffs, we get the following.

$$\begin{aligned}
u_A(\text{offer } \overline{a}) &> u_A(\text{no demand}) \\
\overline{a}(l+1) &> 0
\end{aligned}$$

, which is always true since $l > -1$. Thus, the conditional A always offers $a = \overline{a}$ when $\overline{a} < a^*$.

To sum up, Proposition 1, where the unconditional A does not demand and the conditional A offers $a = \min\{a^*, \overline{a}\}$ and T accepts, happens when Line 21, 22, 24, and 26 are satisfied.

Line 21 is more strict than Line 26 because

$$\begin{aligned}
&(\underline{\beta}+1)a^* > \overline{\beta}(p_h - p_m - c_T) + c_A \\
\Leftrightarrow\ &(\underline{\beta}+1)(p_h - p_l) - \overline{\beta}(p_h - p_m - c_T) > c_A
\end{aligned} \tag{28}$$

Given the range of c_A in Line 8 in Assumption, we need to show that the maximum value of c_A is smaller than the left-hand side of Line 28.

$$\begin{aligned}
&(\underline{\beta}+1)(p_h - p_l) - \overline{\beta}(p_h - p_m - c_T) > \overline{\beta}(p_m - p_l) \\
\Leftrightarrow\ &(p_h - p_l)(\underline{\beta}+1-\overline{\beta}) > 0 > -\overline{\beta}c_T
\end{aligned}$$

$$
\begin{aligned}
&(\underline{\beta}+1)(p_h-p_l)-\overline{\beta}(p_h-p_m-c_T)>\underline{\beta}(p_h-p_l)\\
&\Leftrightarrow p_h-p_l>0
\end{aligned}
$$

Both of them are sure to be true. Thus, Line 21 is more strict than Line 26.

Therefore, the set of conditions necessary for Proposition 1 are Line 21, 22, and 24, which are

$$
\begin{aligned}
&l<0\\
&\overline{\beta}(p_h-p_m-c_T)+c_A>\pi>\underline{\beta}(p_h-p_l)
\end{aligned}
$$

As a final step, we need to check the existence of π.

$$
\begin{aligned}
&\overline{\beta}(p_h-p_m-c_T)+c_A>\underline{\beta}(p_h-p_l)\\
&\Leftrightarrow \overline{\beta}>\frac{\underline{\beta}(p_h-p_l)-c_A}{p_h-p_m-c_T}
\end{aligned}
\tag{29}
$$

Then, let's check if this satisfies $1>\overline{\beta}>0$.

The numerator in Line 29, $\underline{\beta}(p_h-p_l)-c_A$, is positive because of the condition of c_A (Line 8 in Assumption). So, in order to satisfy $\overline{\beta}>0$, the denominator in Line 29 must be positive.

$$
\begin{aligned}
p_h-p_m-c_T&>0\\
p_h-p_m&>c_T
\end{aligned}
\tag{30}
$$

In order to satisfy $1>\overline{\beta}$, we need to make sure the following condition is true.

$$
\begin{aligned}
1&>\frac{\underline{\beta}(p_h-p_l)-c_A}{p_h-p_m-c_T}\\
&\Leftrightarrow \frac{p_h-p_m-c_T+c_A}{p_h-p_l}>\underline{\beta}
\end{aligned}
\tag{31}
$$

This needs to be in the range of $0<\underline{\beta}<1$. First, this satisfies $\underline{\beta}>0$ since the numerator is positive thanks to $p_l-p_m>c_T$ (Line 30) and the denominator is also positive for sure

$(p_h > p_l)$. Second, this satisfies $\underline{\beta} < 1$ because

$$\begin{aligned} & \frac{p_h - p_m - c_T + c_A}{p_h - p_l} < 1 \\ \Leftrightarrow\ & p_m - p_l + c_T > c_A \\ \Leftrightarrow\ & p_m - p_l + c_T > \overline{\beta}(p_m - p_l) > c_A \end{aligned}$$

This is sure to be true because of the upper limit of c_A in Line 8 in Assumption.

Therefore, Proposition 1 exists when Line 21, 22, 24, 29, 30, and 31 are satisfied. □

Proof of Proposition 2

Proposition 2 is a separating equilibrium where the unconditional A offers $a = \overline{a}$ and stays in the alliance after T's rejection, and the conditional A offers $a = a^*$ and (in the off-equilibrium path) A withdraws from the alliance after T's rejection. T with the unconditional A rejects any cost-sharing offer and T with the conditional A accepts $a \leq a^*$.

Most of the proof of this equilibrium is offered in the previous proof. Actors' strategies in the crisis bargaining are discussed and set in Assumption and the previous proof. In the cost-sharig renegotiation stage, the unconditional A does not withdraw from the alliance after T's rejection of the cost-sharing offer when Line 21 is satisfied, and the conditional A withdraws from the alliance when Line 22 is satisfied. π exists between these two conditions when Line 29, 30, and 31 are met. T's strategy of acceptance or rejection does not change, either. The unconditional A offers $a = \overline{a}$ when Line 25 is satisfied. The conditional A offers $a = a^*$ when $\overline{a} > a^*$ and Line 27 are satisfied.

The only thing we need to additionally prove is the existence of l in Line 27. The existence of l in $l^* = \frac{(\underline{\beta}+1)a^* - \pi}{\overline{a} - a^*} > l > 0$ should be checked.

$$\begin{aligned} & l^* > 0 \\ \Leftrightarrow\ & \frac{(\underline{\beta}+1)a^* - \pi}{\overline{a} - a^*} > 0 \\ \Leftrightarrow\ & (\underline{\beta}+1)a^* > \pi \quad (\because \overline{a} > a^*) \end{aligned}$$

The maximum value of π is $\overline{\beta}(p_h - p_m - c_T) + c_A > \pi$ as shown in Line 21, so we need to show that π's maximum value is smaller than $(\underline{\beta}+1)a^*$.

$$
\begin{aligned}
&(\underline{\beta}+1)a^* > \overline{\beta}(p_h - p_m - c_T) + c_A \\
\Leftrightarrow\ &(\underline{\beta}+1)(p_{h-p_l}) - \overline{\beta}(p_h - p_m - c_T) > c_A
\end{aligned}
$$

This is sure to be true because of the range of c_A in Assumption (Line 8), as shown in Line 28.

Therefore, Proposition 2 exists when Line 21, 22, 25, 27, 29, 30, 31, and $\overline{a} > a^*$. □

Proof of Proposition 3

Proposition 3 is a separating equilibrium where both the committed A and the uncommitted A offer $\overline{a}$, the committed A does not withdraw from the alliance, and the uncommitted A withdraws. T rejects the cost-sharing offer regardless of the type of A.

Again, most of the proof is already shown in Proof of Proposition 1. The unconditional A does not withdraw from the alliance after T rejects the cost-sharing offer when Line 21 is satisfied, and the conditional A withdraws from the alliance when Line 22 is satisfied. π exists between these two conditions when Line 29, 30, and 31 are met. T's strategy of acceptance or rejection of the cost-sharing offer is also the same as the previous two equilibria. The committed A offers $a = \overline{a}$ when Line 25 is satisfied.

The difference from previous equilibria is the cost-sharing offer by the conditional type of A. When T's maximum ability to pay is larger than a^* ($\overline{a} > a^*$) and and $l > 0$ as Line 25 requires, the conditional A's payoffs are

$$
\begin{aligned}
u_A(\text{no demand}) &= \underline{\beta}(p_h - c_T) - \pi \\
\max_a u_A(\text{offer } a \le a^*) &= u_A(\text{offer } a = a^*) \quad (\because l > 0) \\
&= \underline{\beta}(p_h - c_T) - \pi + a^* + la^* \\
\max_a u_A(\text{offer } a > a^*) &= \max_a \underline{\beta}(p_l - c_T) + la \quad (\because l > 0) \\
&= u_A(\text{offer } a = \overline{a}) = \underline{\beta}(p_l - c_T) + l\overline{a}
\end{aligned}
$$

A offers $\overline{a}$ if

$$
\begin{aligned}
&u_A(\text{offer } a = \overline{a}) > u_A(\text{no demand}) \\
&\Leftrightarrow \underline{\beta}(p_l - c_T) + l\overline{a} > \underline{\beta}(p_h - c_T) - \pi \\
&\Leftrightarrow \pi > \underline{\beta}(p_h - p_l) - l\overline{a}
\end{aligned}
$$

, which is sure to be true because of the lower limit of π in Line 22 ($\pi > \underline{\beta}(p_h - p_l)$).

Also,

$$
\begin{aligned}
&u_A(\text{offer } a = \overline{a}) > u_A(\text{offer } a = a^*) \\
&\Leftrightarrow \underline{\beta}(p_l - c_T) + l\overline{a} > \underline{\beta}(p_h - c_T) - \pi + a^* + la^* \\
&\Leftrightarrow l > l^* = \frac{(\underline{\beta}+1)a^* - \pi}{\overline{a} - a^*}
\end{aligned} \tag{32}
$$

Because of Line 25, we should check if this is within the range of $0 < l$. $0 < l^*$ is true as proved in the previous section.

Therefore, Proposition 3 exists when Line 21, 22, 29, 30, and 32. □

Proof of Proposition 4

Proposition 4 is a pooling equilibrium where both the unconditional and conditional types do not make a cost-sharing demand. In the off-equilibrium path, T would reject any offers regardless of A's type, and both types of A do not withdraw from the alliance after the rejection. On this off-equilibrium path, C believes that the probability that A is the unconditional type after its withdrawal from the alliance is less than d^*.

First of all, let's consider C's offer in the crisis bargaining if A withdraws from the alliance. As discussed at the beginning of the proof of Proposition 1, C offers $x = p_m - c_T$ when A is unconditional and $x = p_l - c_T$ when A is conditional. When C is not sure about A's type, C has to decide to make a risky offer ($x = p_l - c_T$) or a safer offer ($x = p_m - c_T$). Let d be C's belief of $Pr(\overline{\beta}|A\text{'s withdrawal})$. The payoffs of each offer for C are

$$
\begin{aligned}
&u_C(x = p_m - c_T) = 1 - p_m + c_T \\
&u_C(x = p_l - c_T) = d(1 - p_m - c_C) + (1 - d)(1 - p_l + c_T) \\
&u_C(x = p_m - c_T) < u_C(x = p_l - c_T) \\
&\Leftrightarrow d < d^* = \frac{p_m - p_l}{p_m - p_l + c_T + c_C}
\end{aligned} \tag{33}
$$

Thus, C offers $x = p_m - c_T$ iff $d \geq d^*$ and $x = p_l - c_T$ iff $d < d^*$.

Hereafter, I assume that C's off-equilibrium-path belief that A is unconditional after she withdraws from the alliance is less than d^*, or $0 < d < d^*$, which means that C offers the risky offer, $x = p_l - c_T$, after alliance termination.

With this belief in mind, let's consider actors' strategies in the cost-sharing negotiation stage.

First, if A remains in the alliance after T's rejection, C offers $x = p_h - c_T$ and this is accepted by T. A's payoff in such a situation is

$$u_A(\text{stay}) = \beta(p_h - c_T) - \pi + la$$

When A withdraws from the alliance, C offers $x = p_l - c_T$ as $d < d^*$. T with the unconditional A rejects this offer and war happens. T with the conditional A accepts the offer, and peace is sustained. Thus, A's payoffs of withdrawing from the alliance are

$$u_A(\text{withdraw}) = \begin{cases} \overline{\beta} p_m - c_A + la & (\text{When } A \text{ is unconditional}) \\ \underline{\beta}(p_l - c_T) + la & (\text{When } A \text{ is conditional}) \end{cases}$$

The unconditional A remains in the alliance when

$$\begin{aligned} & u_A(\text{stay}) > u_A(\text{withdraw}) \\ \Leftrightarrow\ & \overline{\beta}(p_h - p_m - c_T) + c_A > \pi \end{aligned} \tag{34}$$

The conditional A also remains in the alliance when

$$\begin{aligned} & u_A(\text{stay}) > u_A(\text{withdraw}) \\ \Leftrightarrow\ & \underline{\beta}(p_h - p_l) > \pi \end{aligned} \tag{35}$$

Line 35 is more strict than Line 34 when

$$\begin{aligned} \overline{\beta}(p_h - p_m - c_T) + c_A > \underline{\beta}(p_h - p_l) \\ \Leftrightarrow \overline{\beta} > \frac{\underline{\beta}(p_h - p_l) - c_T}{p_h - p_m - c_T} \end{aligned} \tag{36}$$

This satisfies $0 < \overline{\beta} < 1$ when

$$\frac{p_h - p_m - c_T + c_A}{p_h - p_l} > \underline{\beta} \tag{37}$$

$$p_h - p_m > c_T \tag{38}$$

(See Line 30 and 31 in Proof of Proposition 1)

Next, regardless of A's type, T does not have an incentive to accept any cost-sharing offer.

$$u_T(\text{accept}) = p_h - c_T - a$$
$$u_T(\text{reject}) = p_h - c_T$$

$u_T(\text{reject}) > u_T(\text{accept})$ is sure be the case because $a > 0$.

The next step is to investigate the optimal level of A's cost-sharing offer. A's payoffs are expressed as follows.

$$u_A(\text{no demand}) = \beta(p_h - c_T) - \pi$$
$$u_A(\text{offer } a) = \beta(p_h - c_T) - \pi + la$$

A makes "no demand" when

$$l < 0 \tag{39}$$

A offers $a = \overline{a}$ because A gets the largest payoff when

$$l > 0 \tag{40}$$

To sum up, when Line 33, 35, 36, 37, and 38 are satisfied, both the unconditional A and the conditional A do not withdraw from the alliance after the rejection by T. When Line 39 is satisfied, both types of A do not offer anything. Therefore, when these conditions are met, Proposition 4 exists.

Finally, I check the Intuitive Criterion proposed by Cho and Kreps (1987). After A's withdrawal from the alliance, C's belief is $d < d^*$. This off-equilibrium-path belief fails to pass the Intuitive Criterion if the conditional A would not choose to withdraw from the alliance even when C believes that A is the unconditional type after withdrawal and thus makes a generous offer. If this is the case, the unconditional type of A can credibly say that A withdrawing from the alliance is the unconditional type, and this is inconsistent with $d < d^*$.

To check this, let's hypothetically assume that C's belief after A's withdrawal is $d = 1$, meaning that C thinks that A is unconditional after the withdrawal. On such occation, the

conditional type of A's payoffs are

$$u_A(\text{withdraw}) = \underline{\beta}(p_h - p_l) + la$$
$$u_A(\text{stay}) = \underline{\beta}(p_h - p_l) - \pi + la$$

The conditional type of A surely withdraws from the alliance since $\pi > 0$. Thus, Proposition 4 satisfies the Intuitive Criterion.

□

Proof of Proposition 5

The necessary parts of the proof of Proposition 5 are shown in the previous proof.

When Line 33, 35, 36, 37, and 38 are satisfied, both the unconditional and conditional A do not withdraw from the alliance after the rejection by T. When Line 40 is satisfied, both types of A offer $a = \overline{a}$. Therefore, when these conditions are met, Proposition 5 exists. This equilibrium also satisfies the Intuitive Criterion for the same reason above. □